The wave equation

1. Suppose we have a square room with one wall at 80˚C, an adjacent wall at 60˚C and the other two walls at 20˚C. What will the temperature distribution be throughout the room be if break each wall into three intervals? The unknowns are indicated by u_1 through u_4 indicating the ordering of the linear equations in the answers.

80	80	80	80
20	u_1	u_2	60
20	u_3	u_4	60
20	20	20	60

Answer:

```
>> [4 -1 -1 0; -1 4 0 -1; -1 0 4 -1; 0 -1 -1 4] \ [100 140 40 80]'
```

80	80	80	80
20	47.5	57.5	60
20	32.5	42.5	60
20	20	20	60

2. Suppose that the wall opposite the hottest wall is insulated. How does this change the temperatures?

Answer:

```
>> [4 -1 -1 0; -1 4 0 -1; -1 0 3 -1; 0 -1 -1 3] \ [100 140 20 60]'
```

80	80	80	80
20	50.737	61.263	60
20	41.684	54.316	60
20	*	*	60

3. Suppose that the hottest wall is replaced with one that is also insulated. Without reworking the mathematics, what do you expect the temperature distribution to be throughout the room?

Answer:

```
>> [3 -1 -1 0; -1 3 0 -1; -1 0 3 -1; 0 -1 -1 3] \ [20 60 20 60]'
```

*	*	*	*
20	33.333	46.667	60
20	33.333	46.667	60
*	*	*	*

4. Suppose you have a twisting hallway that has insulated walls, but one end of the hallway is in contact with an exterior door that is at 1˚C and the other is one that is kept at a warm room temperature at 25˚C. Assume that the hallway is not heated in any way. What is the temperature throughout the hallway?

*	25	*	*	*	*	*	*
*	u_1	*	u_{13}	u_{14}	u_{15}	u_{16}	*
*	u_2	*	u_{12}	*	*	u_{17}	*
*	u_3	*	u_{11}	*	u_{19}	u_{18}	*
*	u_4	*	u_{10}	*	u_{20}	*	*
*	u_5	*	u_9	*	u_{21}	u_{22}	*
*	u_6	u_7	u_8	*	*	u_{23}	*
*	*	*	*	*	*	1	*

Answer:

```
>> A = 2*diag(ones(23,1)) - diag(ones(22,1),1) - diag(ones(22,1),-1);
>> b = zeros(23,1);
>> b(1) = 25; b(end) = 1;
>> A \ b
ans =
   24.0000
   23.0000
   22.0000
   21.0000
   20.0000
   19.0000
   18.0000
   17.0000
   16.0000
   15.0000
   14.0000
   13.0000
   12.0000
   11.0000
   10.0000
    9.0000
    8.0000
    7.0000
    6.0000
    5.0000
    4.0000
    3.0000
    2.0000
```

5. Does the answer in the last question make sense?

Answer: Yes, as you go away from the warmer door, the temperature should drop along the length of the hallway, even if it meanders.